VOLUME 78 SEPARATE No. D-51

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

JANUARY, 1952

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

DISCUSSION OF LATERALLY LOADED PLANE STRUCTURES AND STRUCTURES CURVED IN SPACE

(*Published in January, 1951*)

By Uku Müllersdorf, Maurice Barron, and Frank Baron and James P. Michalos

STRUCTURAL DIVISION

Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

PRICE $0.50 PER COPY

GUIDEPOST FOR TECHNICAL READERS

"Proceedings-Separates" of value or significance to readers in various fields are here listed, for convenience, in terms of the Society's Technical Divisions. Where there seems to be an overlapping of interest between Divisions, the same Separate number may appear under more than one item.

Technical Division	*Proceedings-Separate Number*
Air Transport	42, 43, 48, 52, 60, 93, 94, 95, 100, 103, 104, 108 (Discussion: D-XXVIII, D-7, D-16, D-18, D-23, D-43)
City Planning	58, 60, 62, 64, 93, 94. 99, 101, 104, 105 (Discussion: D-16, D-23, D-43, D-60, D-62)
Construction	43, 50, 55, 71, 92, 94, 103, 108, 109, 113 (Discussion: D-3, D-8, D-17, D-23, D-36, D-40)
Engineering Economics	46, 47, 62, 64, 65, 68, 69, 95, 100, 104 (Discussion: D-2, D-19, D-27, D-30, D-36, D-57)
Engineering Mechanics	41, 49, 51, 54, 56, 59, 61, 88, 89, 96 (Discussion: D-5, D-XXIII, D-XXV, D-18, D-24, D-33, D-34 D-49, D-54, D-61)
Highway	43, 44, 48, 58, 70, 100, 105, 108, 113 (Discussion: D-XXVIII, D-7, D-13, D-16, D-23, D-60)
Hydraulics	50, 55, 56, 57, 70, 71, 78, 79, 80, 83, 86, 92, 96, 106, 107, 110, 111, 112, 113 (Discussion: D-XXVII, D-9, D-11, D-19, D-28, D-29)
Irrigation	46, 47, 48, 55, 56, 57, 67, 70, 71, 87, 88, 90, 91, 96, 97, 98, 99, 102, 106, 109, 110, 111, 112, 114 (Discussion: D-XXIII, D-3, D-7, D-11, D-17, D-19, D-25-K, D-29, D-30, D-38, D-40, D-44, D-47, D-57)
Power	48, 55, 56, 69, 71, 88, 96, 103, 106, 109, 110 (Discussion: D-XXIII, D-2, D-3, D-7, D-11, D-17, D-19, D-25-K, D-30, D-38, D-40, D-44)
Sanitary Engineering	55, 56, 87, 91, 96, 106, 111 (Discussion: D-10, D-29, D-37, D-60)
Soil Mechanics and Foundations	43, 44, 48, 94, 102, 103, 106, 108, 109 (Discussion: D-4, D-XXVIII, D-7, D-43, D-44)
Structural	42, 49, 51, 53, 54, 59, 61, 66, 89, 100, 103, 109, 113 (Discussion: D-3, D-5, D-8, D-13, D-16, D-17, D-21, D-23, D-24, D-25-K, D-32, D-33, D-34, D-37, D-39, D-42, D-49, D-51, D-54, D-61)
Surveying and Mapping	50, 52, 55, 60, 63, 65, 68 (Discussion: D-60)
Waterways	41, 44, 45, 50, 56, 57, 70, 71, 96, 107, 112, 113 (Discussion: D-8, D-9, D-19, D-27, D-28)

A constant effort is made to supply technical material to Society members, over the entire range of possible interest. Insofar as your specialty may be covered inadequately in the foregoing list, this fact is a gage of the need for your help toward improvement. Those who are planning papers for submission to "Proceedings-Separates" will expedite Division and Committee action measurably by first studying the ASCE "Guide for Development of Proceedings-Separates" as to style, content, and format. For a copy of this Manual, address the Manager, Technical Publications, ASCE, 33 W. 39th Street, New York 18, N. Y.

Published at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

DISCUSSION

Uku Müllersdorf[21].—The ingenious method of calculation described in this paper is of great interest. In the writer's opinion it constitutes a valuable contribution to the science of calculation of complicated structures, especially plane structures submitted to the action of lateral loads.

The method seems to have only one disadvantage, and that is, it implies many purely mechanical calculating operations. As a consequence there is the risk that the designer may, so to say, lose contact with the structure and its mode of action. This may result in possible errors of calculation going undiscovered until a relatively late stage.

As previously stated, the great merit of this method appears to be in connection with relatively complicated structural problems such as arches with variable rigidity. In simpler cases some other method might give a better conspectus and in many cases shorten the calculation. In this connection the method of successive approximations may be suggested to be used in accordance with the same principles that are applied when calculating the action of loads in the principal plane of the structure. The method is particularly suitable if the loads are stationary. In case of moving loads, or if influence line diagrams are to be drawn, the method suggested by the authors will probably be more appropriate. In the latter case other methods, as for instance the method of primary moments suggested by A. Efsen,[22] may also be contemplated. Originally presented as a method of solving plane problems, with certain amplifications it may also be used for the type of loads investigated by the authors. The writer has used this method in several instances, once even in the investigation of an arch structure in which the influence of the deck structure also was taken into consideration. The constants necessary for the use of this method are first calculated and thereafter used wherever loads occur. This method, as well as the method of successive approximations, may of course be applied to continuous systems.

Concerning arches, it is to be noted that by applying loads (especially if the structure is subjected to lateral loads) a combined action is established between the arch and the deck structure. As the latter generally has a great lateral rigidity, a considerable part of the load is transferred by it. However, in this case the arch may be considered as the fundamental system, and it would be very appropriate, in cases in which great accuracy is required, to calculate this system by the method suggested in the paper. In any special case, however, the lateral deformations must not give rise to perceptible additional moments, as theoretically such a system is variable in reference to lateral as well as to vertical loads. If a deformation occurs perpendicular to the principal plane of the structure, additional moments are caused even by loads in the plane of the system, just as a possible oblique position of the columns between the deck structure and the arch may give rise to additional lateral loads.

Note.—This paper by Frank Baron and James P. Michalos was published in January, 1951, as *Proceedings-Separate No. 51*. The numbering of footnotes in this separate is a continuation of the consecutive numbering used in the original paper.

[21] Structural Engr., Stockholm, Sweden.

[22] "Die Methode der primären Momente," by A. Efsen, Copenhagen, Denmark, 1931.

MAURICE BARRON,[23] M. ASCE.—Under the heading "A General Procedure for Structures Curved in Space," the authors outline a broad application of their procedure which includes "* * * any shape in space and any variation in cross section along its length * * *" and "* * * subjected to loads in any direction and to moments about any axis." This is a very broad application which covers all space structures.

The skewed rigid-frame bridge and the skewed arch are space structures and they are presumably included in the category of space structures which the authors state can be analyzed by their proposed method. It is proper, then, to compare the author's proposed method with the large amount of published material available for solid barrel skewed structures.

The writer initially calls attention to the theory, equations, and method of analysis proposed in 1924[24] by J. Charles Rathbun, M. ASCE. The counterpart of each stress and each redundant in Eq. 12, and the elastic equations, Eq. 15, may be found by such a comparison. Therefore, all the research,[25] test data,[26,27] samples of design, and the limitations, which have been collected over 30 years may be used to properly evaluate the proposed methods.

One important observation is immediately apparent. The authors assume three axes in space which are mutually perpendicular (as does Mr. Rathbun). Elastic equations (the same as those proposed by Mr. Rathbun) are then written and solved simultaneously to determine the redundant reactions. Although this procedure is theoretically correct, it has been shown[28,29] that the elastic system about each axis may be treated independently. Furthermore, the elastic system about the X-axis is a primary one, and the elastic systems about the other two axes are of such a lower order that it is impossible to solve the equations simultaneously with any degree of confidence.[30] The difficulty encountered due to small differences may not hold for the idealized problem which the authors present, but the writer has found[31] that the more idealized the problem, the greater is the difference in order between the primary elastic system and the secondary system. For idealized problems most of the effects of the secondary system on the primary system vanish completely. The initiated analyst will recognize this important fact as a principle instead of parts of equations which become insignificant (or vanish entirely) in the solution of several simultaneous equations.

---

[23] Farkas & Barron, Cons. Engrs., New York, N. Y.

[24] "Analysis of the Stresses in the Ring of a Concrete Skew Arch," by J. Charles Rathbun, *Transactions*, ASCE, Vol. LXXXVII, 1924, p. 611.

[25] "Effect of Skew Angle on Rigid-Frame Reactions," by Walter C. Boyer, *Proceedings-Separate No. 32*, ASCE, September, 1950.

[26] "An Experimental Study of the Reactions of a Two-Span Skewed Rigid Frame Bridge," by Gordon B. Fisher, thesis presented to The Johns Hopkins University, at Baltimore, Md., in partial fulfilment of the requirements for a degree in 1948.

[27] "The Use of Models in the Solution of Indeterminate Structures," by George E. Beggs, *Journal of the Franklin Institute*, March, 1927, p. 375.

[28] "Reinforced Concrete Skewed Rigid-Frame and Arch Bridges," by Maurice Barron, *Proceedings-Separate No. 13*, ASCE, April, 1950.

[29] "The Rigid-Frame Bridge," by Arthur G. Hayden and Maurice Barron, John Wiley & Sons, Inc., New York, N. Y., 1950.

[30] Discussion by Leo Sos of "Reinforced Concrete Skewed Rigid-Frame and Arch Bridges," by Maurice Barron, *Proceedings-Separate No. D-13*, April, 1951, p. 13.

[31] Discussion by Maurice Barron of "Effect of Skew Angle on Rigid-Frame Reactions," by Walter C. Boyer, *Proceedings-Separate No. D-32*, June, 1951, p. 2.

Frank Baron,[32] Assoc. M. ASCE, and James P. Michalos,[33] Assoc. M. ASCE.—The writers appreciate the constructive discussions of the paper. Mr. Müllersdorf has raised several questions of concern to designers. He seems to be disturbed that the procedure advocated in the paper apparently has the disadvantage of implying many purely mechanical computations. This disadvantage does not exist; and, contrary to Mr. Müllerdorf's belief, the procedure permits the designer to retain the picture of structural action. The procedure is flexible and meets the requirements of the designer as well as those of the analyst. It lends itself to the conduct of either informal qualitative studies of structural behavior or of more formal quantitative studies. For this reason, particular attention has been given to the possible interpretations of the various terms appearing in the formal statement of the analytical procedure.

In addition, the results are interpreted in terms of a pressure line concept. A procedure of sketching, similar to that used for an arch with loads in the plane of the arch,[19,20] may be developed for drawing approximate diagrams of moments about the x, y, and z axes of laterally loaded plane structures and structures curved in space. Such a procedure has been indicated in the discussions of Figs. 6, 7, and 12. It consists of drawing the curves of moments in two installments. First, draw a statically possible distribution of moments about the x, y, and z axes of the structure. Such a distribution may be shown in three separate diagrams, each diagram being associated with an axis. Then draw a planar distribution of correction moments and check the requirements of geometry. A planar distribution of correction moments is represented by three straight lines, one line in each of the above diagrams. The requirements of geometry are satisfied if the angle changes associated with the moments are balanced. Additional illustrations of moments drawn in this way may be inspected elsewhere.[34]

As noted by Mr. Müllersdorf, the procedure can be used in obtaining constants required in methods of successive approximations for analyzing continuous structures. The procedure can also be used in studies of arch structures in which the influence of the deck structure is to be taken into account. For such cases, the additional requirements of geometry must be considered. In certain cases the procedure can be used in obtaining the additional moments caused by changes in the dimensions of a structure. The additional moments can be obtained by a numerical method of successive approximations provided the computations converge and are not prohibitive in number.

The writers are not in agreement with Mr. Baron's interpretation of the general procedure for structures curved in space nor with his conclusions concerning the sensitivity of computations for such structures as indicated by previous studies of skewed rigid-frame bridges or skewed arches.

---

[32] Prof., Dept. of Civ. Eng., Technological Inst., Northwestern Univ., Evanston, Ill.

[33] Associate Prof., Dept. of Civ. Eng., Iowa State College, Ames, Iowa.

[19] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1932, p. 289.

[20] *Ibid.*, p. 295.

[34] "Effects of Lateral Loads on Arches," by James P. Michalos, *Journal*, American Concrete Institute, Vol. 22, January, 1951, p. 377.

Mr. Baron calls attention to the theory, equations, and methods of analysis proposed in 1924[25] by J. Charles Rathbun, M. ASCE, for the analysis of skewed rigid frames and arches. Although a reasonable facsimile of each stress and each redundant in Eq. 12 and Eq. 15 may be found in corresponding equations of Mr. Rathbun's paper, the resemblance is purely coincidental and ceases at that stage. The facsimile occurs since in each paper six equations of statics and six equations of geometry are written. Mr. Baron's remarks concerning the resemblance are subject to possible misinterpretations. The writers did not set up six simultaneous equations and solve them solely to obtain the redundants in a skewed frame or arch.

It is repeated that the general procedure of the paper can be used for determining moments and shears in a structure curved or segmental in space and continuous between two supports. The structure may have any shape in space, any variation in cross section along its length, and it may be subjected to loads in any direction and moments about any axis. The procedure is general, and for structures lying in a plane it reduces to the Hardy Cross column analogy or to the shear and torsion analogy of the paper.

If objections to the assumptions and limitations listed by Mr. Rathbun for the analysis of skewed frames are removed and if the relationships between moments and angle changes for a skewed element are considered acceptable, the skewed rigid-frame and the skewed arch can be classified as special examples of "structures curved in space." In that case, the concepts and tabular forms of the writers' general procedure are applicable to the analysis of such structures.

As previously stated, the writers have not concerned themselves with uncertainties in the relationships between statics and geometry caused by the characteristics of a material. It was assumed that the properties necessary for computing the relationships between statics and geometry were known. For the use of the general procedure in the analysis of skewed rigid frames or arches, it is then desirable to summarize the acceptable relationships between moments and angle changes of skewed elements in space.

Mr. Baron's further remarks leave the impression that several problems of algebra remain in obtaining solutions to the six simultaneous equations of geometry. It is emphasized that the philosophy and the various concepts introduced in the paper are independent of the precise analytical procedure. The concept of the pressure line and the concept that permits the interchange of computations of geometry with those of statics were employed by the authors in stating, restating, and obtaining an algebraic solution to the six complicated equations of geometry. It is noted, however, that the computer's task has been simplified to a purely arithmetical task. In addition, the designer's task has been simplified. For his needs as well as those of the computer, each term of the analytical procedure has been interpreted and restated. But by means of the concepts introduced in the paper, the designer should be enabled to visualize the important aspects of structural action, to inspect the important variables, and, if desired, to obtain quick and reasonable estimates of quantities

[25] "Effect of Skew Angle on Rigid-Frame Reactions," by Walter C. Boyer, *Proceedings-Separate No. 32*, ASCE, September, 1950.

involved without the necessity of conducting lengthy arithmetical computations.

The problem of algebra concerning the sensitivity of computations caused by small differences in the coefficients of the six simultaneous equations does not exist in the sense nor to the degree indicated by Mr. Baron. It is unfortunate that Mr. Baron's conclusions concerning the sensitivity of such computations have been extrapolated from experience with the analysis of skewed frames or arches. In connection with this point the following is emphasized:

1. The precise analysis is purely arithmetical. In the analysis any convenient set of rectangular axes can be chosen and any statically possible distribution of moments can first be assumed. It should be obvious that the better the guess, the smaller the correction.
2. The numerical example in Fig. 12 is not at all an idealized problem. It is a bent in space, such as may be necessary for supporting urban elevated freeway construction. It is observed that bending about all axes is important.
3. The skewed frame or arch is, because of its relatively big width of deck, the special problem. Consequently, greatly unbalanced stiffness ratios exist. The effect of such unbalanced ratios on the $d\,a$ values and on succeeding computations can be seen by inspection of the tabular form. The inspection would indicate which operations could be neglected without materially affecting the results. This only emphasizes the versatility of this method of analysis.

In conclusion, the analytical procedures can be formal or informal. They can be numerical or pictorial and can be exact or approximate, depending on the needs of the person conducting the analysis.

AMERICAN SOCIETY OF CIVIL ENGINEERS

OFFICERS FOR 1952

PRESIDENT
CARLTON S. PROCTOR

VICE-PRESIDENTS

Term expires January, 1952:
FRED C. SCOBEY
ALBERT HAERTLEIN

Term expires October, 1952:
WILLIAM R. GLIDDEN
DANIEL V. TERRELL

DIRECTORS

Term expires January, 1952:
WALDO G. BOWMAN
HAROLD L. BLAKESLEE
PAUL L. HOLLAND
EDMUND FRIEDMAN
S. T. HARDING

Term expires January, 1953:
OTTO HOLDEN
FRANK L. WEAVER
GORDON H. BUTLER
G. BROOKS EARNEST
WALTER J. RYAN*
GEORGE W. LAMB

Term expires October, 1952:
MORRIS GOODKIND
MILTON T. WILSON

Term expires October, 1953:
KIRBY SMITH
FRANCIS S. FRIEL
LOUIS R. HOWSON
WALLACE L. CHADWICK
NORMAN R. MOORE
BURTON G. DWYRE

PAST-PRESIDENTS
Members of the Board

ERNEST E. HOWARD
GAIL A. HATHAWAY

TREASURER
CHARLES E. TROUT

EXECUTIVE SECRETARY
WILLIAM N. CAREY

ASSISTANT TREASURER
GEORGE W. BURPEE

ASSISTANT SECRETARY
E. L. CHANDLER

PROCEEDINGS OF THE SOCIETY

SYDNEY WILMOT
Manager of Technical Publications

HAROLD T. LARSEN
Editor of Technical Publications

COMMITTEE ON PUBLICATIONS
WALDO G. BOWMAN

FRANCIS S. FRIEL
S. T. HARDING
OTTO HOLDEN
LOUIS R. HOWSON
NORMAN R. MOORE

* Deceased.